Measurement-Only Topological Quantum Computation

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work done in collaboration with:
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arXiv:0802.0279 (PRL ‘08) and arXiv:0808.1933

Introduction

- Non-Abelian anyons probably exist in certain gapped two dimensional systems:

  - Fractional Quantum Hall Effect (ν=5/2, 12/5, …?)

  - ruthenates, topological insulators, rapidly rotating bose condensates, quantum loop gases/string nets?

- They could have application in quantum computation, providing naturally (“topologically protected”) fault-tolerant hardware.
- Assuming we have them at our disposal, what operations are necessary to implement topological quantum computation?

Anyon Models

(unitary braided tensor categories)

Describe quasiparticle braiding statistics in gapped two dimensional systems.

Finite set $\mathcal{C}$ of anyonic charges: $a, b, c \ldots$

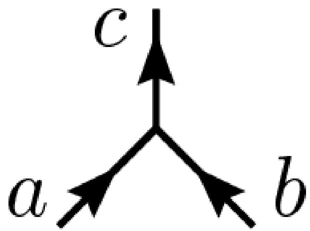
Unique "vacuum" charge, denoted I
has trivial fusion and braiding with all particles.

Fusion rules: $$a \times b = \sum_c N_{ab}^c c$$

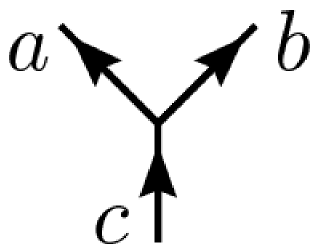
Fusion multiplicities N_{ab}^c are integers specifying the dimension of the fusion and splitting spaces $V_{ab}^c,\ V_c^{ab}$

Hilbert space construct from state vectors associated with fusion/splitting channels of anyons.

Expressed diagrammatically:

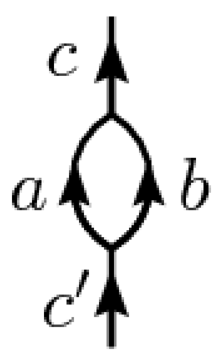


$$= \langle a, b; c| \in V^{c}_{ab}$$

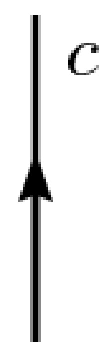


$$= |a, b; c\rangle \in V^{ab}_{c}$$

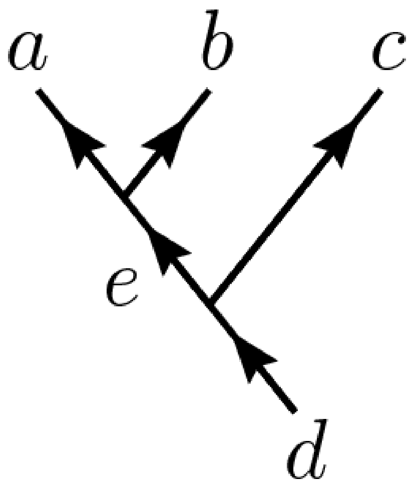
Inner product:



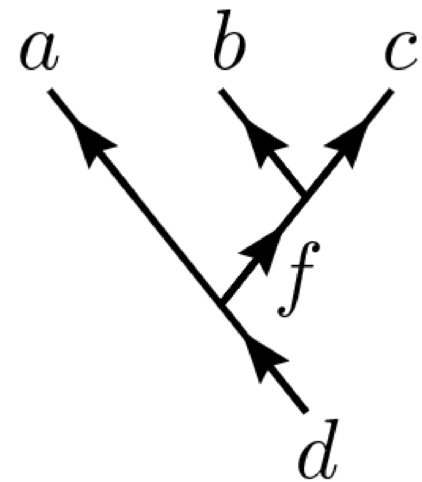
$$= \delta_{cc'}$$



Associativity of fusing/splitting more than two anyons is specified by the unitary F-moves:

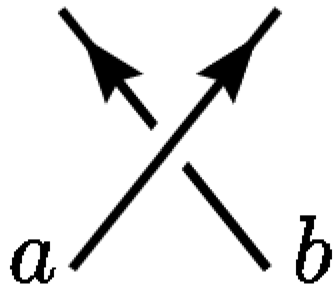


$$= \sum_f \left[F_d^{abc} \right]_{ef}$$

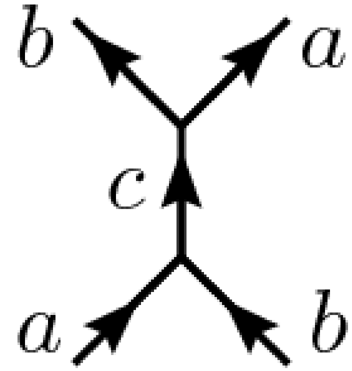


Braiding

$$R^{ab} =$$



$$= \sum_c R_c^{ab}$$



Can be non-Abelian if there are multiple fusion channels c

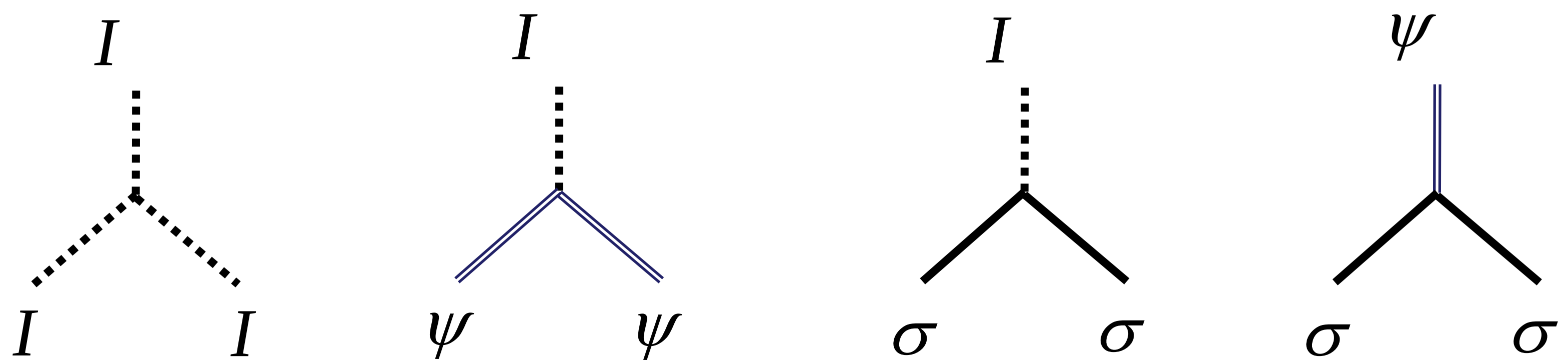
$$|\Psi_\alpha\rangle \mapsto U_{\alpha\beta}[R]|\Psi_\beta\rangle$$

Ising anyons or $SU(2)_2$

- $\nu = \frac{5}{2}$ FQH (Moore-Read `91)
- $\nu = \frac{12}{5}$ and other 2LL FQH? (PB and Slingerland `07)
- Kitaev honeycomb, topological insulators, ruthenates?

Particle types: $I,\ \sigma,\ \psi$ (a.k.a. $0,\ \frac{1}{2},\ 1$)

Fusion rules :

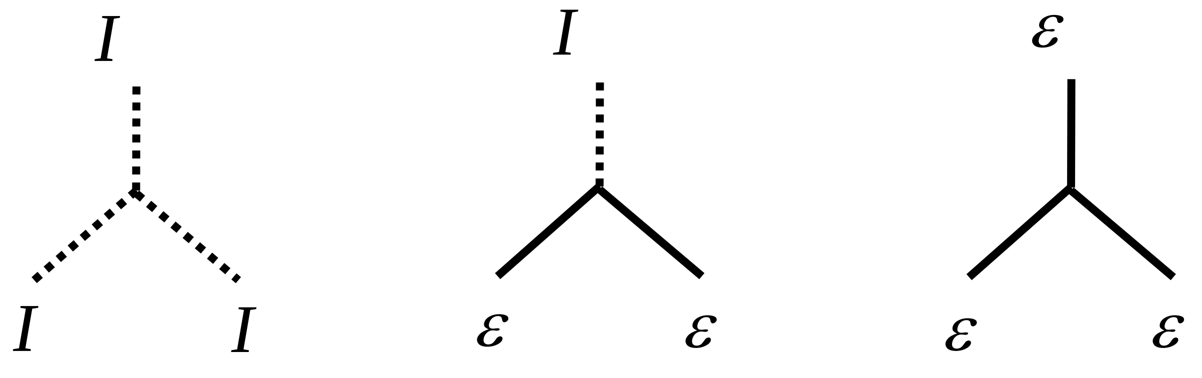


Fibonacci anyons or $SO(3)_3$

- $\nu = \frac{12}{5}$ FQH? (Read - Rezayi `98)
- string nets? (Levin - Wen `04, Fendley et. al. `08)

Particle types: I, ε (a.k.a. 0, 1)

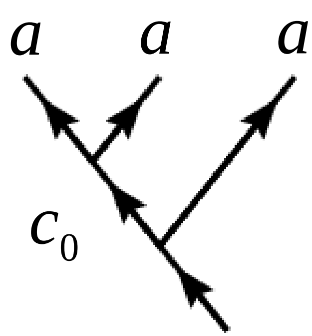
Fusion rules :



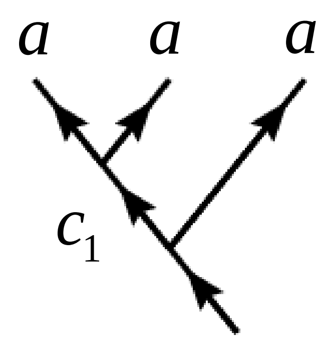
Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)

$|0\rangle \leftrightarrow$



$|1\rangle \leftrightarrow$



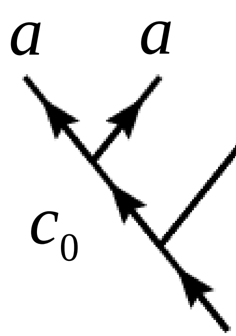
Topological Protection!

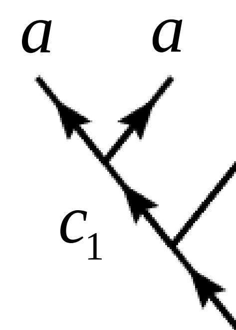
Ising: $a = \sigma,\ c_0 = I,\ c_1 = \psi$

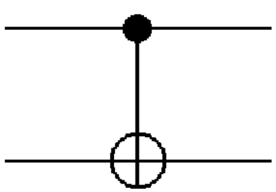
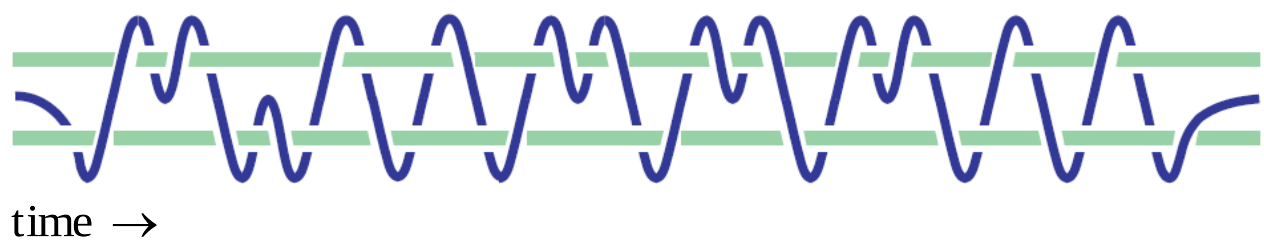
Fib: $a = \varepsilon,\ c_0 = I,\ c_1 = \varepsilon$

Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)

$|0\rangle \leftrightarrow$ 

$|1\rangle \leftrightarrow$ 

 $\leftrightarrow$ 

(Bonesteel, et. al.)

Is braiding computationally universal?

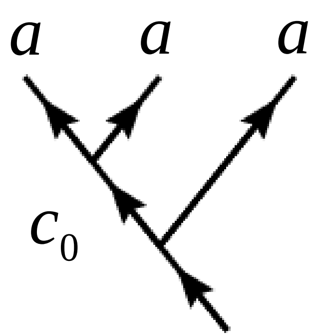
Ising: not quite
(must be supplemented)

Fib: yes!

Topological Quantum Computation

(Kitaev, Preskill, Freedman, Larsen, Wang)

$|0\rangle \leftrightarrow$



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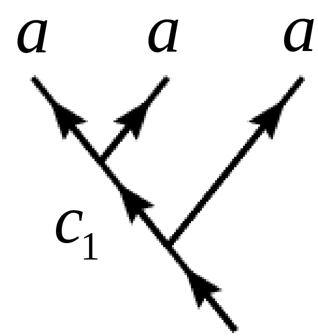
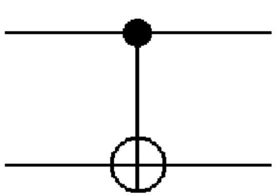
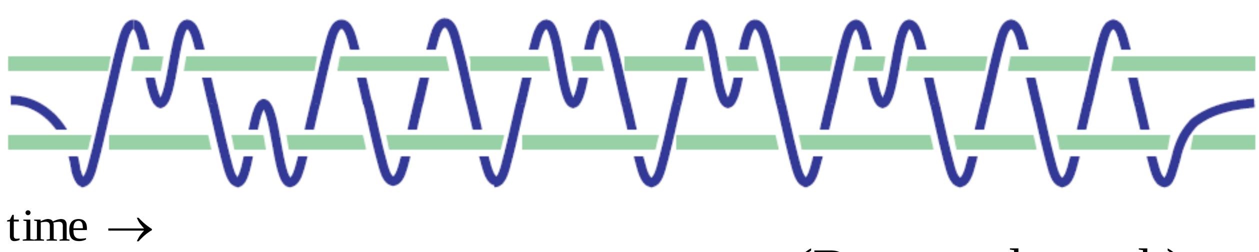
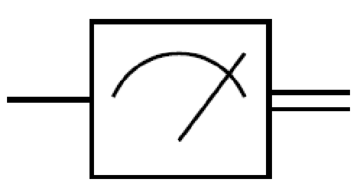




$\leftrightarrow$



(Bonesteel, et. al.)



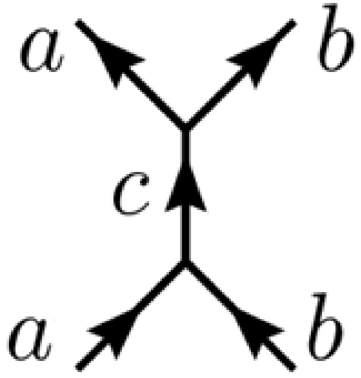
$\leftrightarrow$ Topological Charge Measurement

Topological Charge Measurement

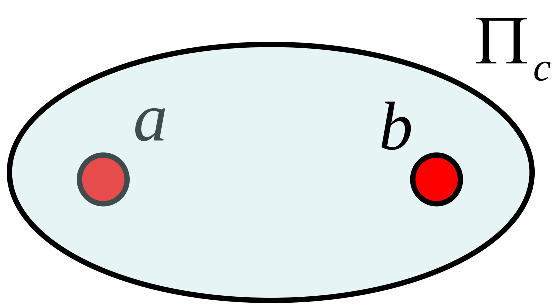
Projective (von Neumann)

e.g. loop operator measurements in lattice models, energy splitting measurement

$$\Pi_c = |a,b;c\rangle\langle a,b;c| =$$



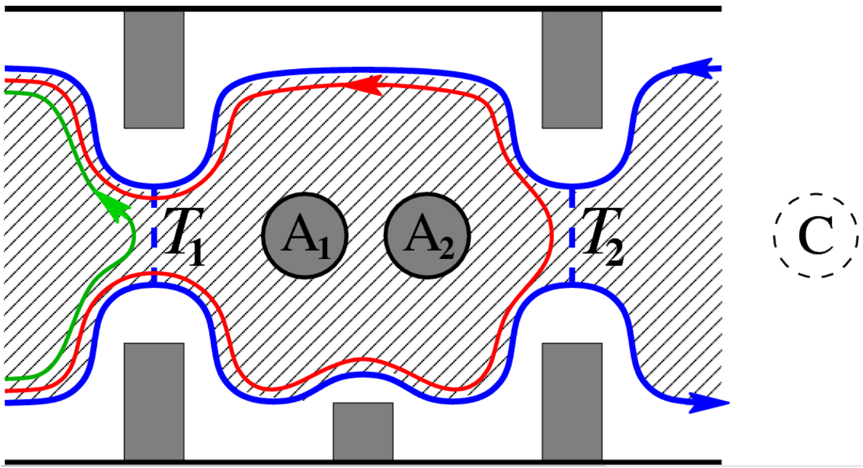
$$|\Psi\rangle \mapsto \frac{\Pi_c|\Psi\rangle}{\langle\Psi|\Pi_c|\Psi\rangle}$$

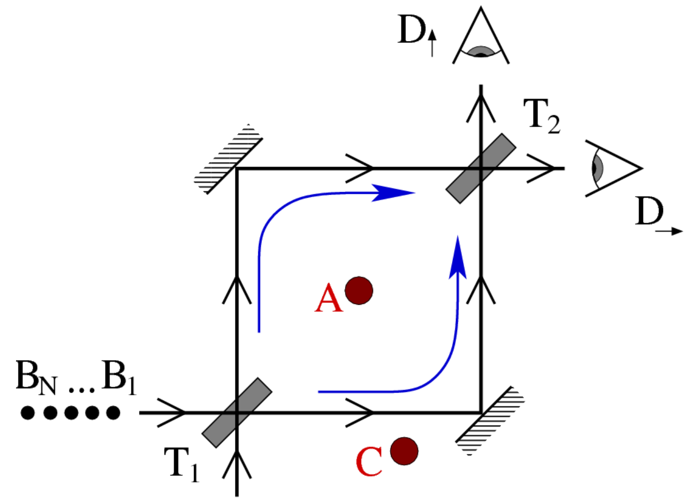


Topological Charge Measurement

Interferometric (PB, Shtengel, Slingerland `07)

e.g. 2PC FQH, and Anyonic Mach-Zehnder (idealized, <u>not</u> FQH)





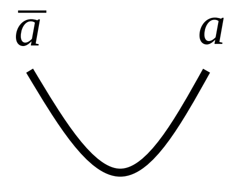
Asymptotically characterized as projection of the target's anyonic charge AND decoherence of anyonic charge entanglement between the interior and exterior of the target region. (more later; ignore for now)

Anyonic State Teleportation

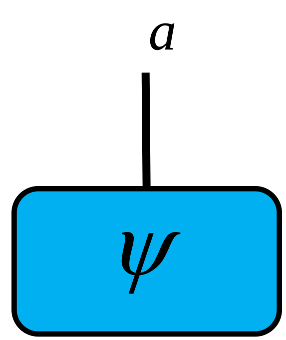
(for projective measurement)

Entanglement Resource: maximally entangled anyon pair

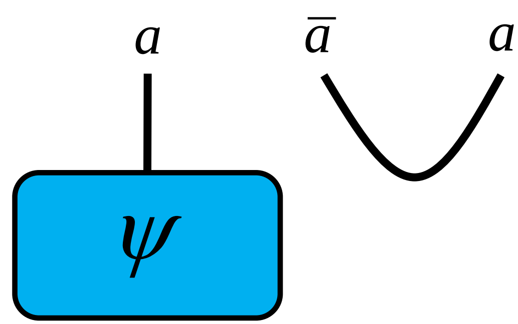
$$\left|\bar{a}, a; I\right\rangle =$$



Want to teleport: $\left|\psi\right\rangle =$



Form: $\left|\psi\right\rangle_1 \left|\bar{a}, a; I\right\rangle_{23} =$



and perform Forced Measurement on anyons 12

Anyonic State Teleportation

Forced Measurement
(projective)

$$\breve{\Pi}_I^{(12)}:$$

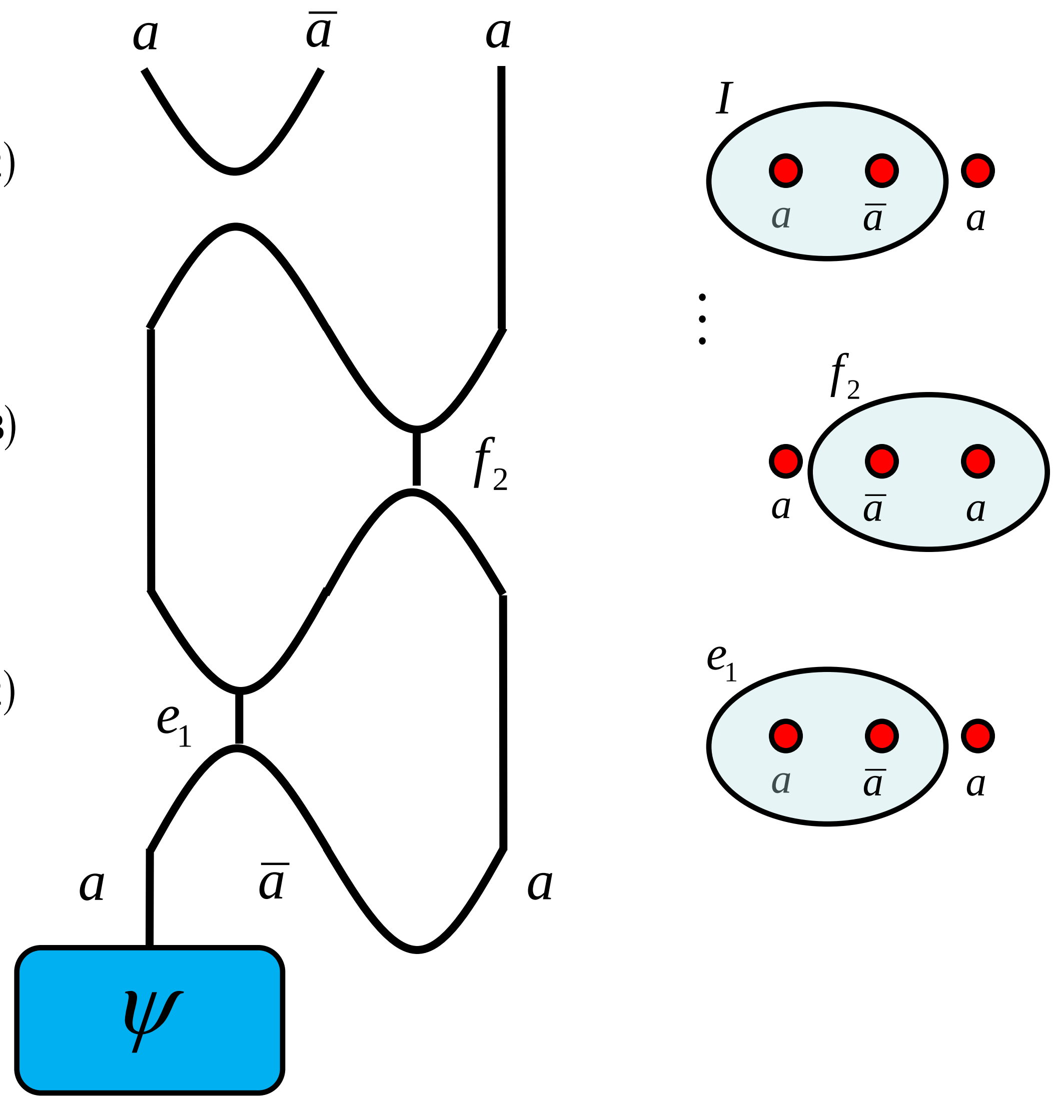
$$\Pi_I^{(12)}$$

$$\vdots$$

$$\Pi_{f_2}^{(23)}$$

$$\Pi_{e_1}^{(12)}$$

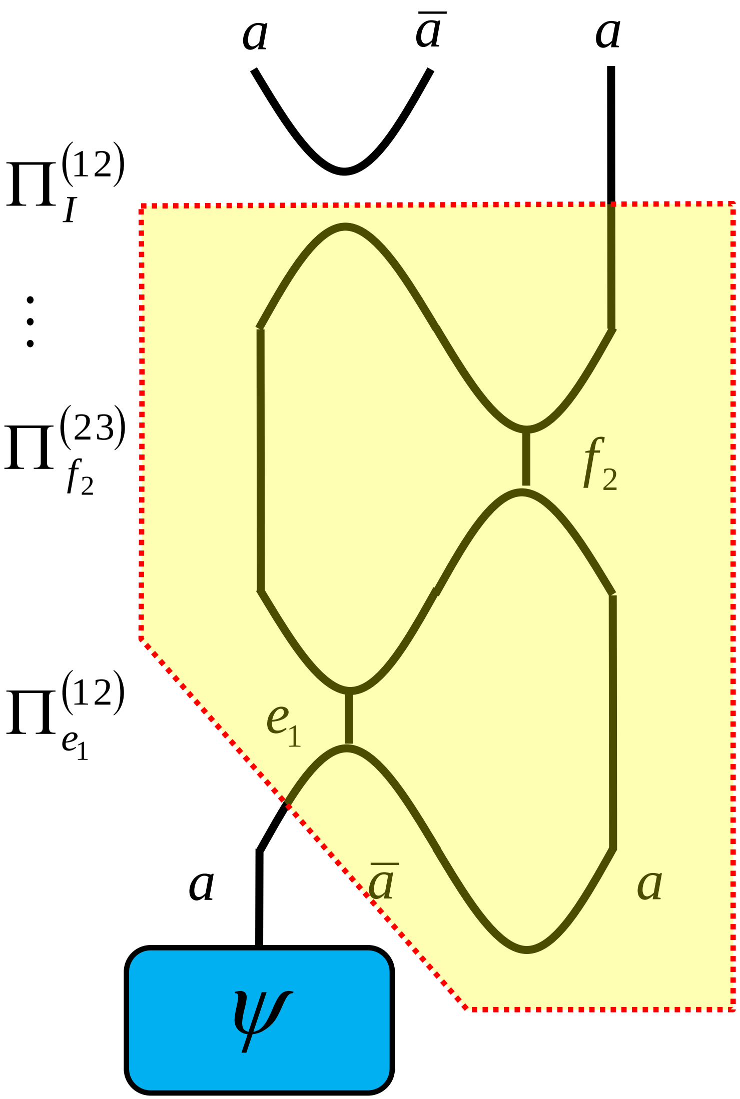
$$|\psi\rangle_1 |\bar{a}, a; I\rangle_{23} =$$



Anyonic State Teleportation

Forced Measurement
(projective)

$$\breve{\Pi}_I^{(12)} :$$

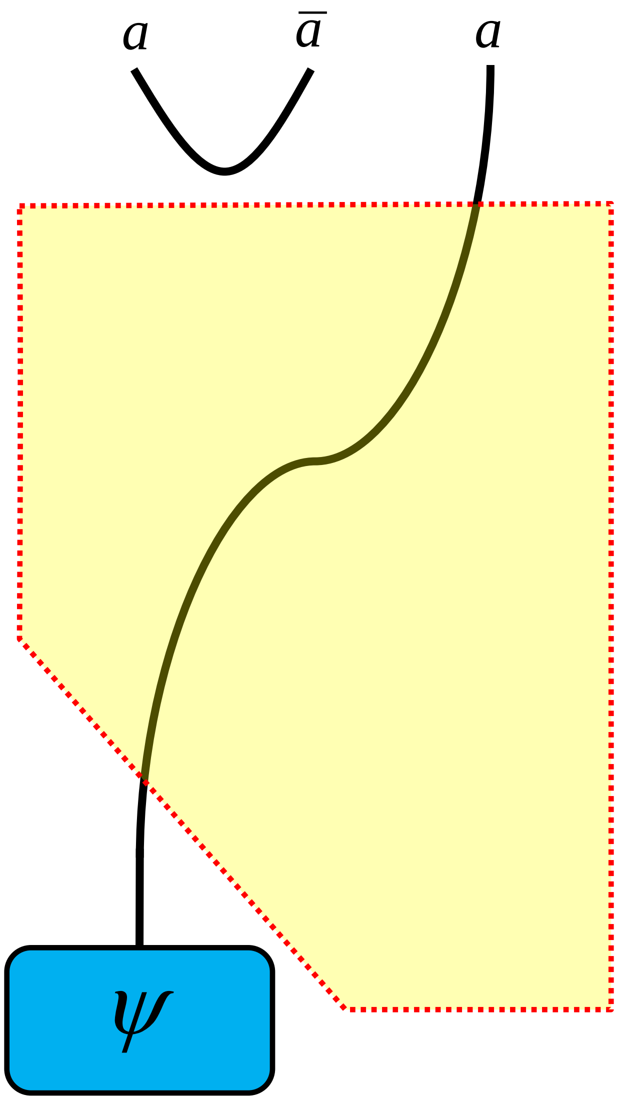


Anyonic State Teleportation

Forced Measurement
(projective)

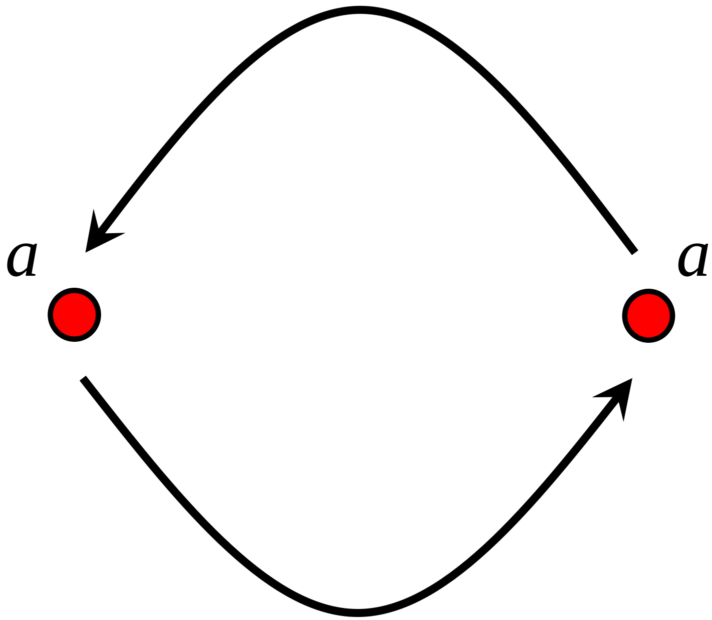
$$\breve{\Pi}_I^{(12)} \cong \Pi_I^{(12)}:$$

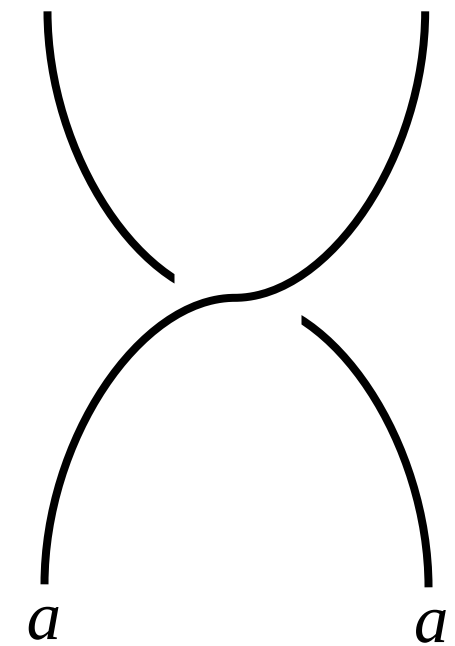
$$\left|\psi\right\rangle_1 \left|\bar{a},a;I\right\rangle_{23}$$
$$\mapsto \left|a,\bar{a};I\right\rangle_{12} \left|\psi\right\rangle_3 =$$



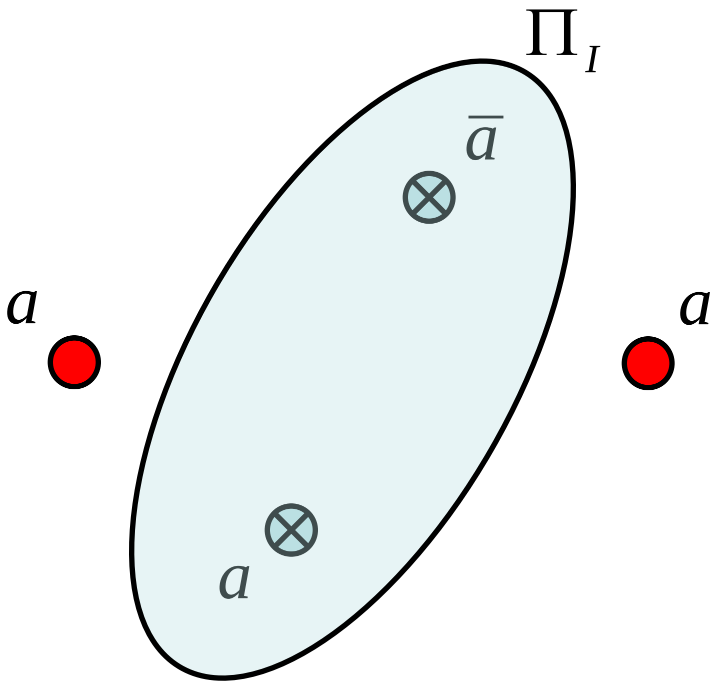
"Success" occurs with probability $\geq \frac{1}{d_a^2}$ for each repeat try.

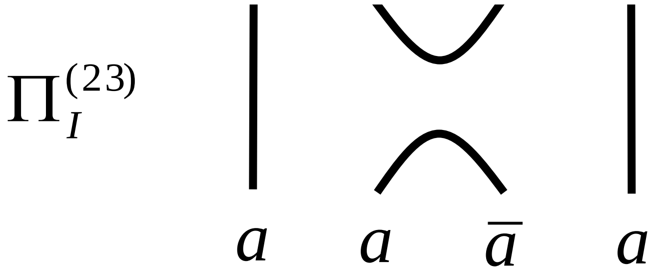
What good is this if we want to braid computational anyons?



$R =$ 

Use a maximally entangled pair and "forced measurements" for a series of teleportations





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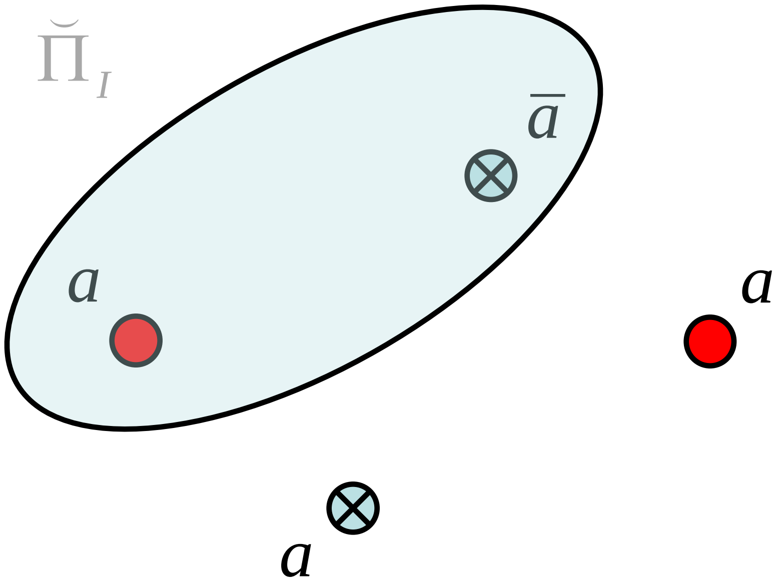
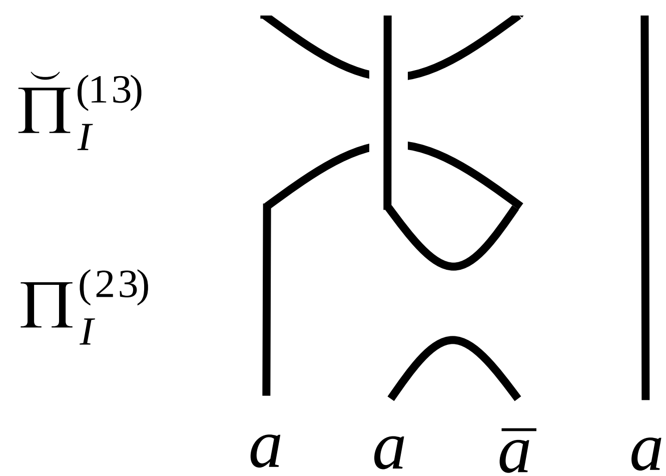
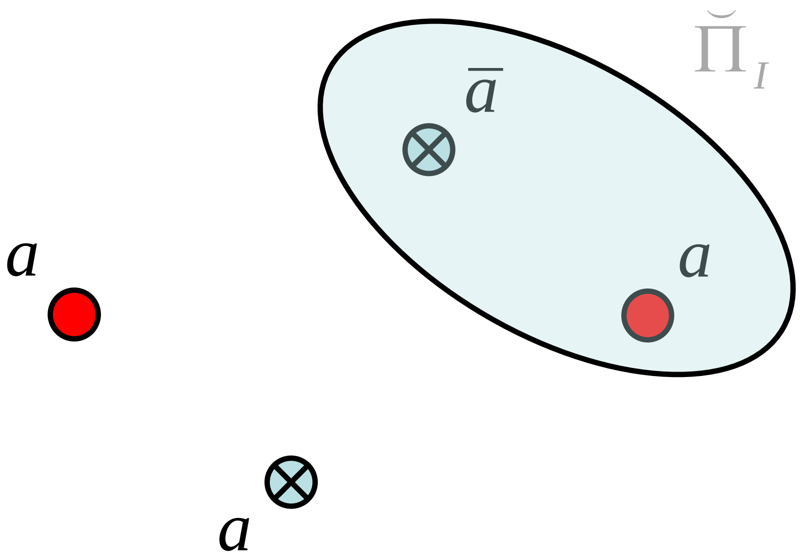
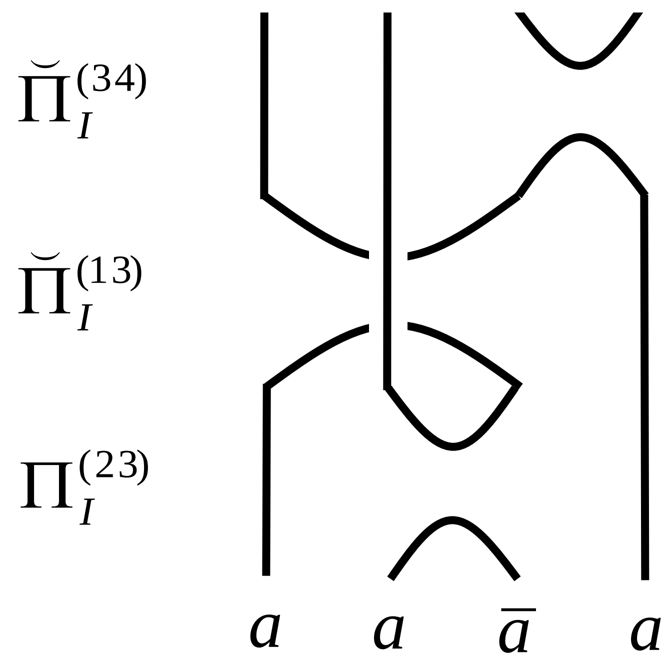




Use a maximally entangled pair and “forced measurements” for a series of teleportations





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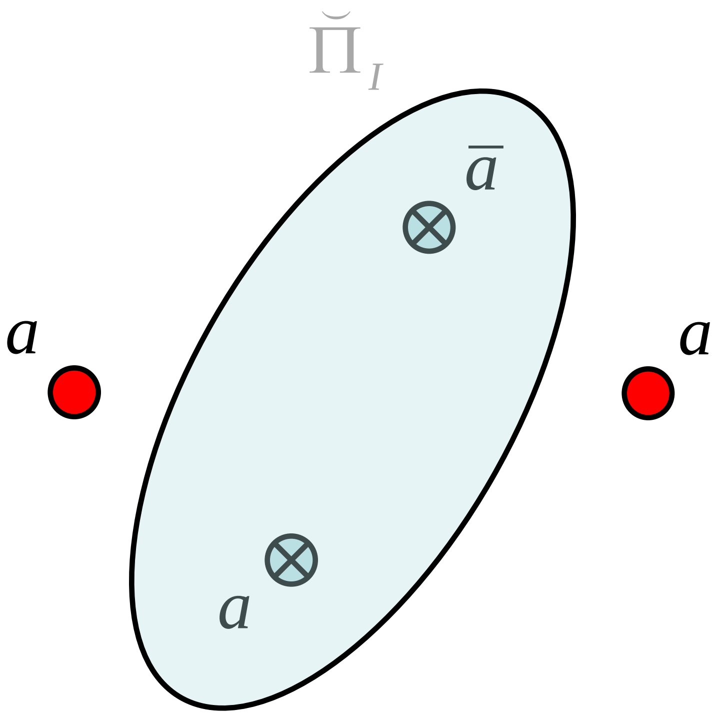
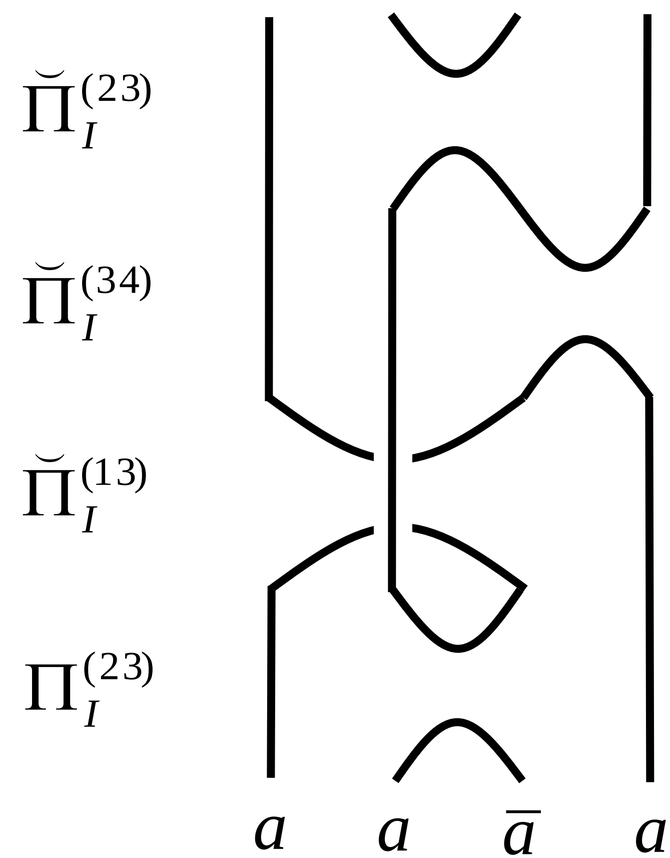
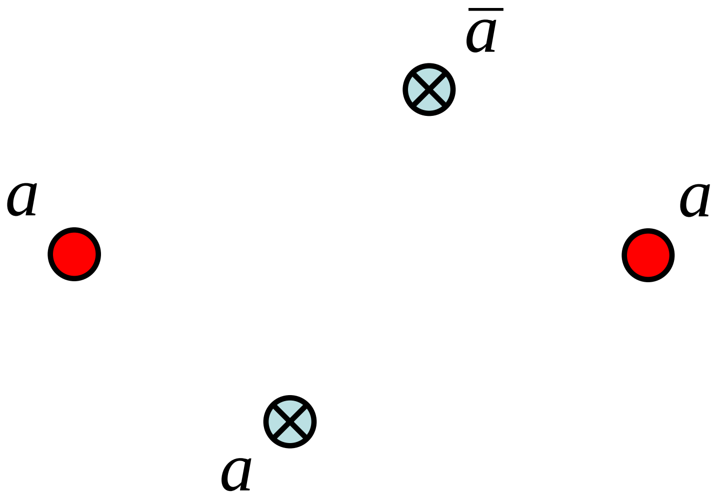
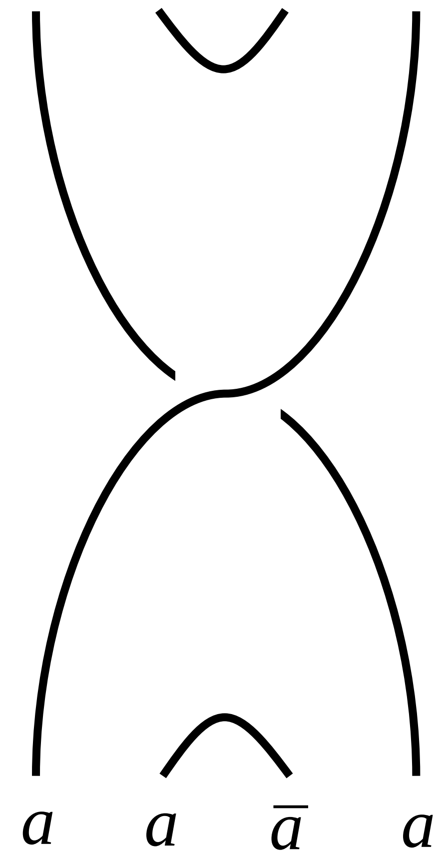




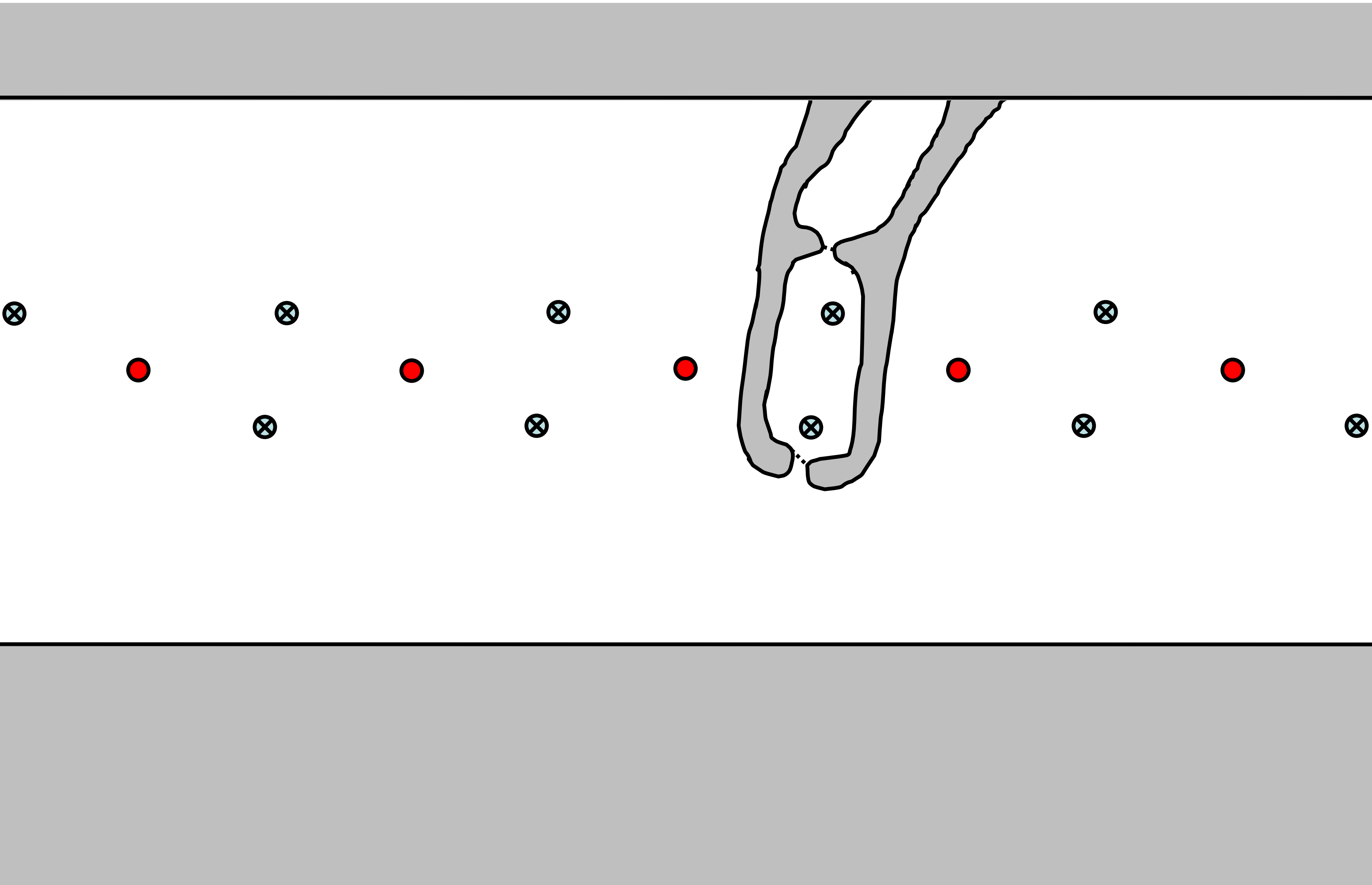
Measurement Simulated Braiding!



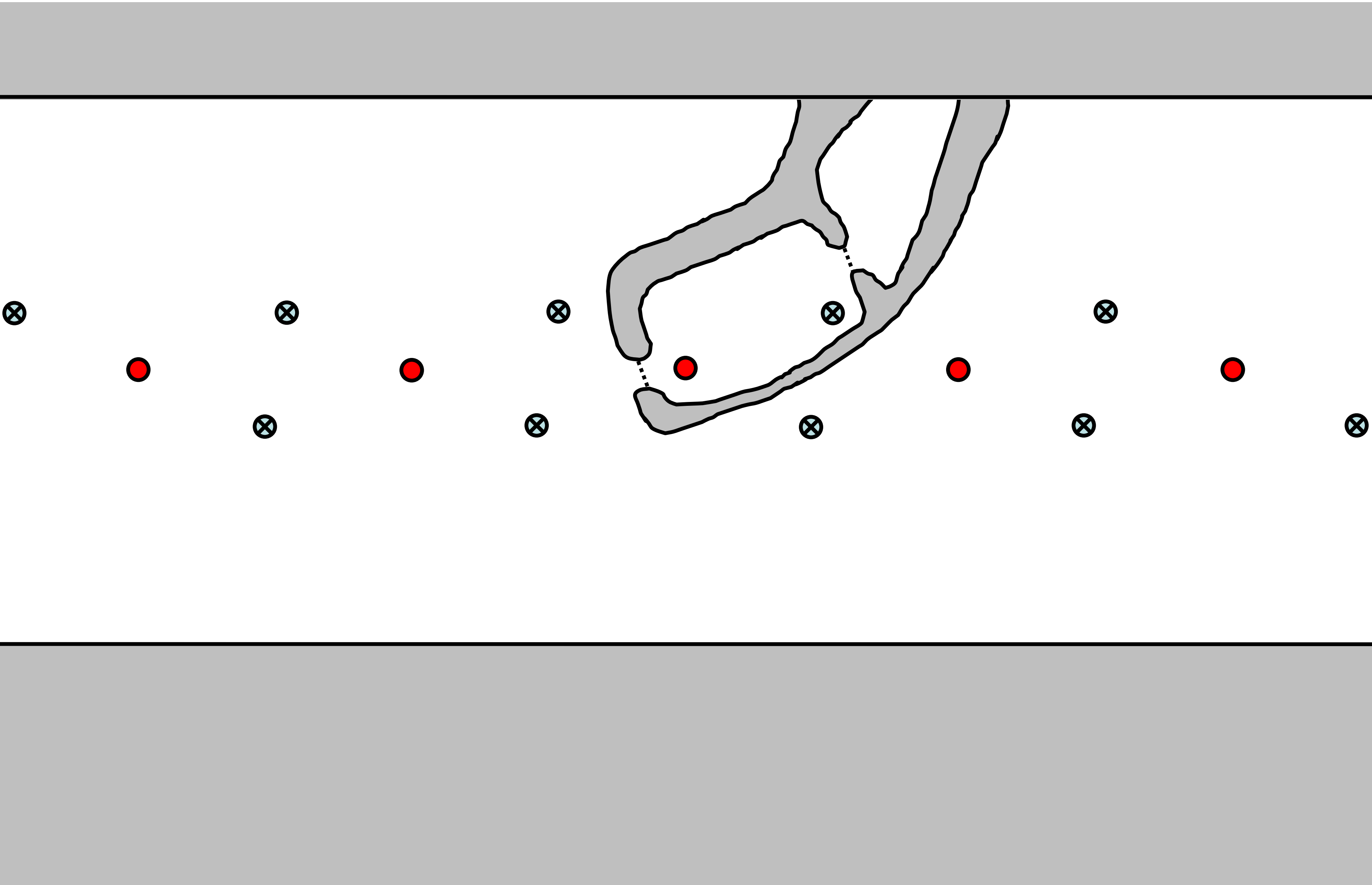
$$R^{(14)} \cong \breve{\Pi}_I^{(23)}\breve{\Pi}_I^{(34)}\breve{\Pi}_I^{(13)}\Pi_I^{(23)} =$$



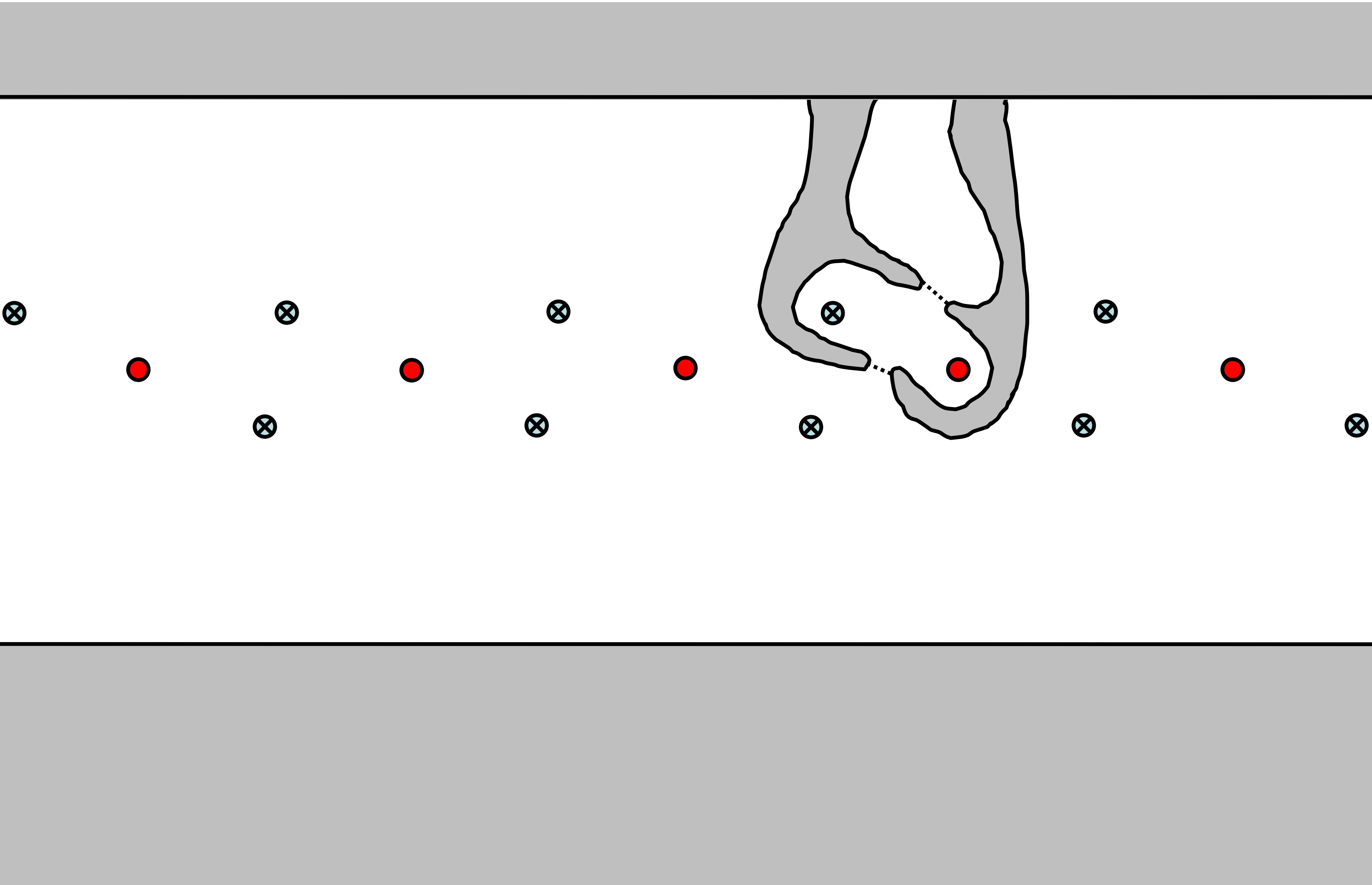
in FQH, for example



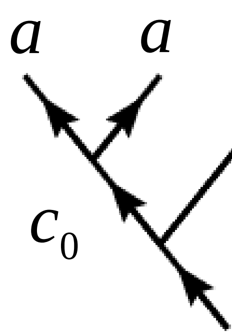
in FQH, for example

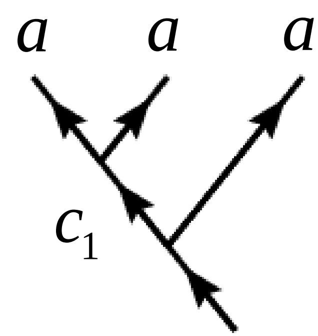


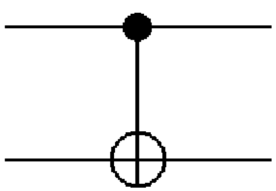
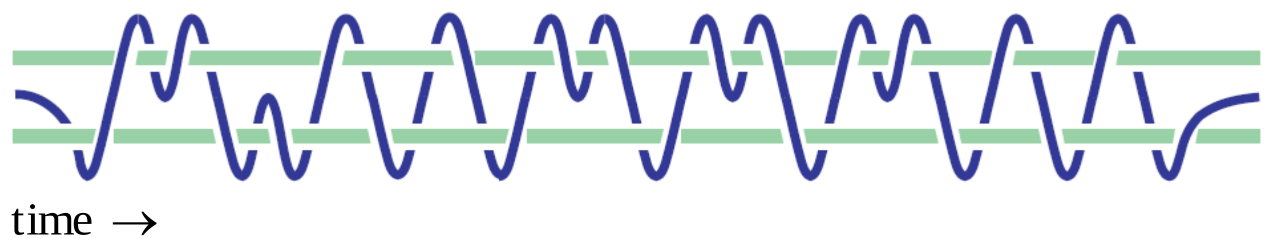
in FQH, for example



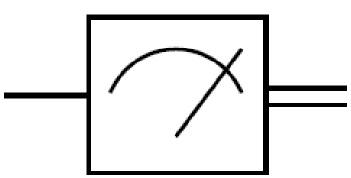
Topological Quantum Computation

$|0\rangle \leftrightarrow$ 

$|1\rangle \leftrightarrow$ 

 $\leftrightarrow$ 

↑ measurement simulated braiding

 $\leftrightarrow$ Topological Charge Measurement

Measurement-Only Topological Quantum Computation

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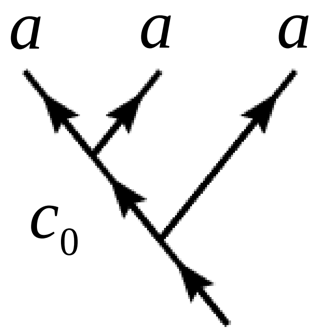
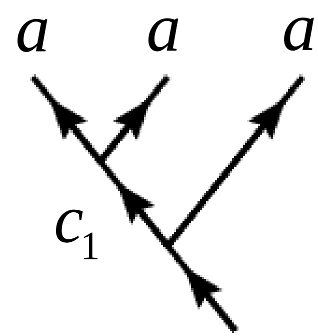
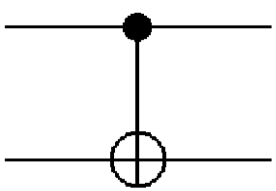
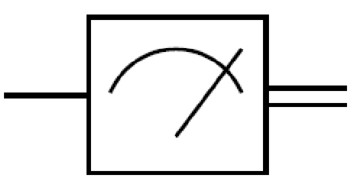


$|1\rangle \leftrightarrow$





$\leftrightarrow$ Topological Charge Measurement

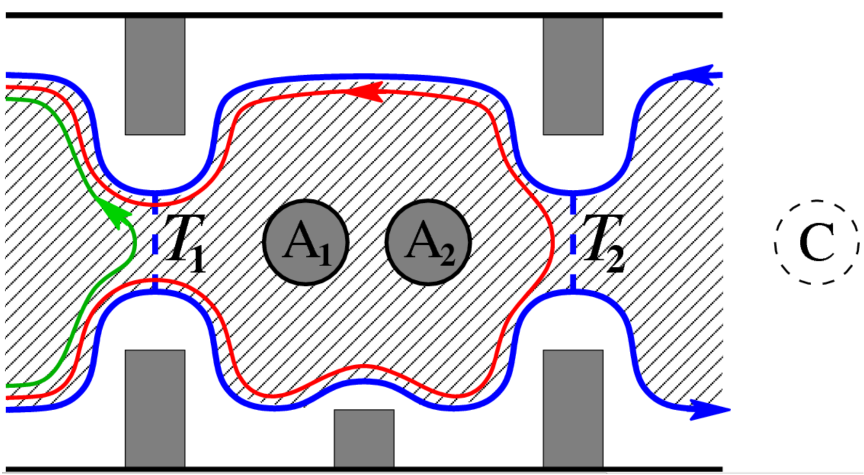


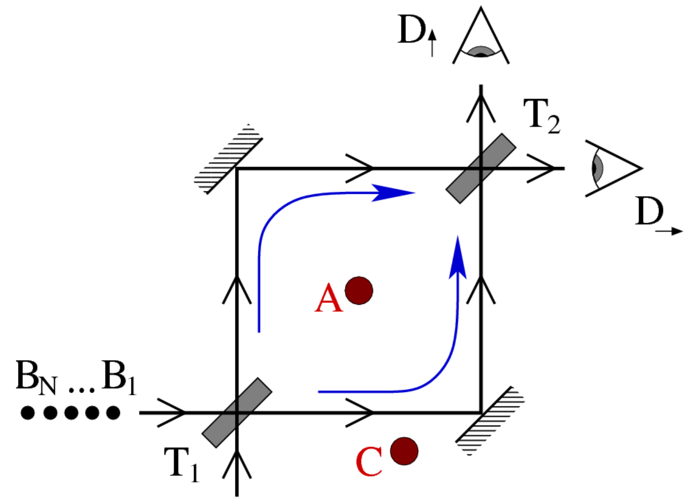
$\leftrightarrow$ Topological Charge Measurement

Topological Charge Measurement

Interferometric (PB, Shtengel, Slingerland `07)

e.g. 2PC FQH, and Anyonic Mach-Zehnder (idealized, not FQH)

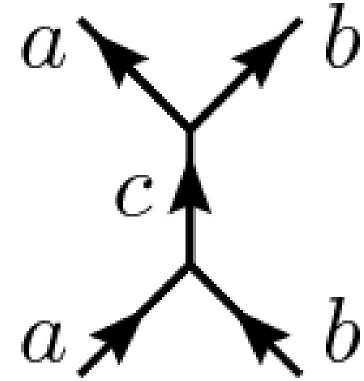




Asymptotically characterized as projection of the target's anyonic charge AND decoherence of anyonic charge entanglement between the interior and exterior of the target region.

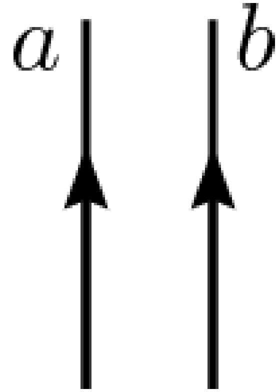
Interferometrical Decoherence of Anyonic Charge Entanglement

$$\rho = |a,b;c\rangle\langle a,b;c| =$$

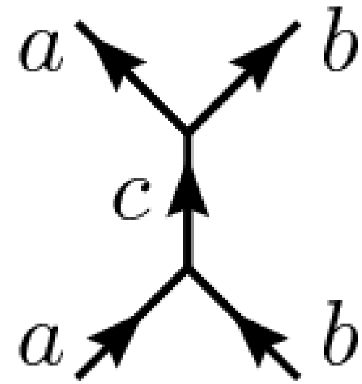


For a inside the interferometer and b outside:

$$D_{\text{int}} : \rho \mapsto$$

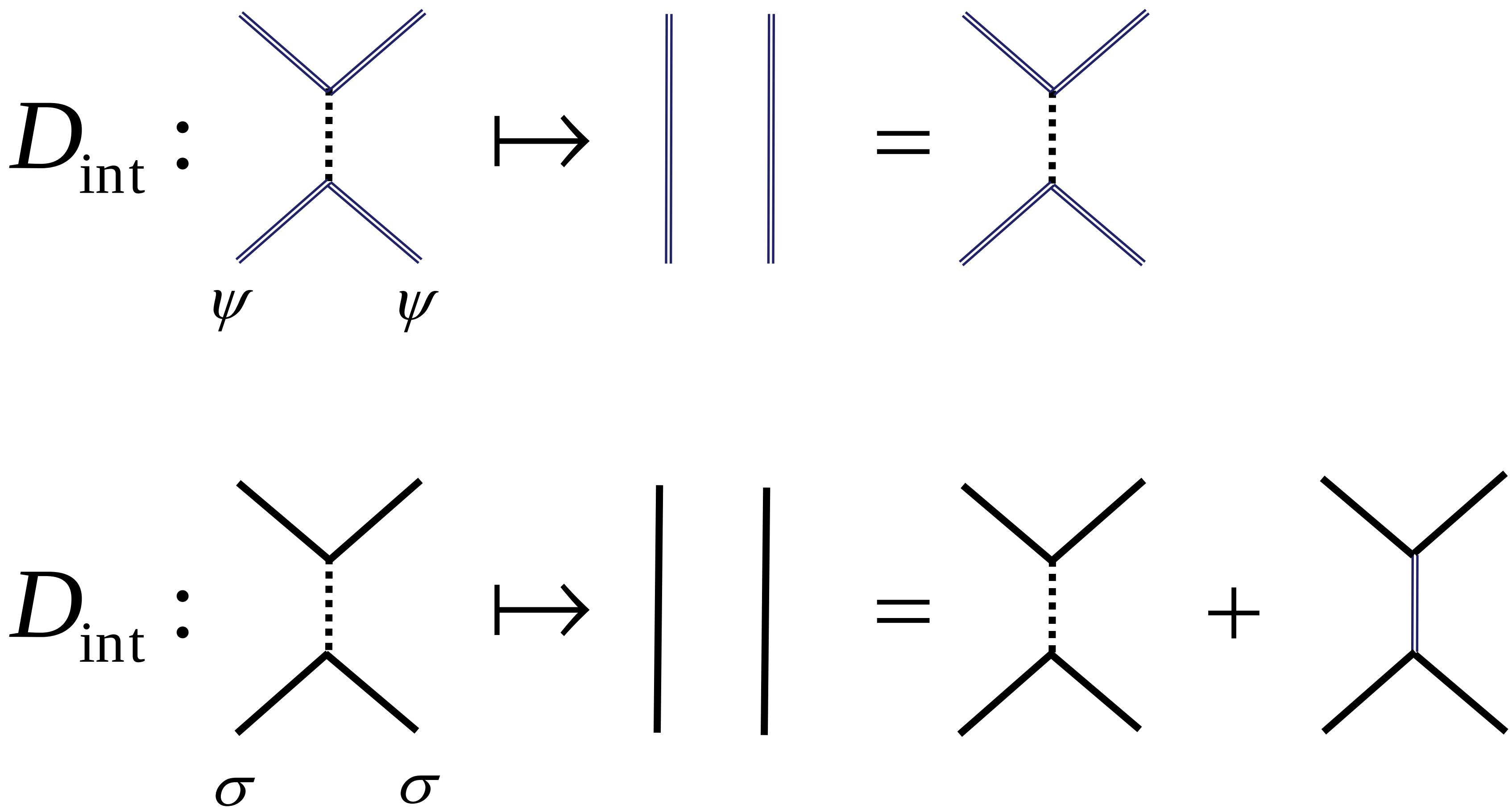


$$= \sum_{c}$$



Interferometrical Decoherence

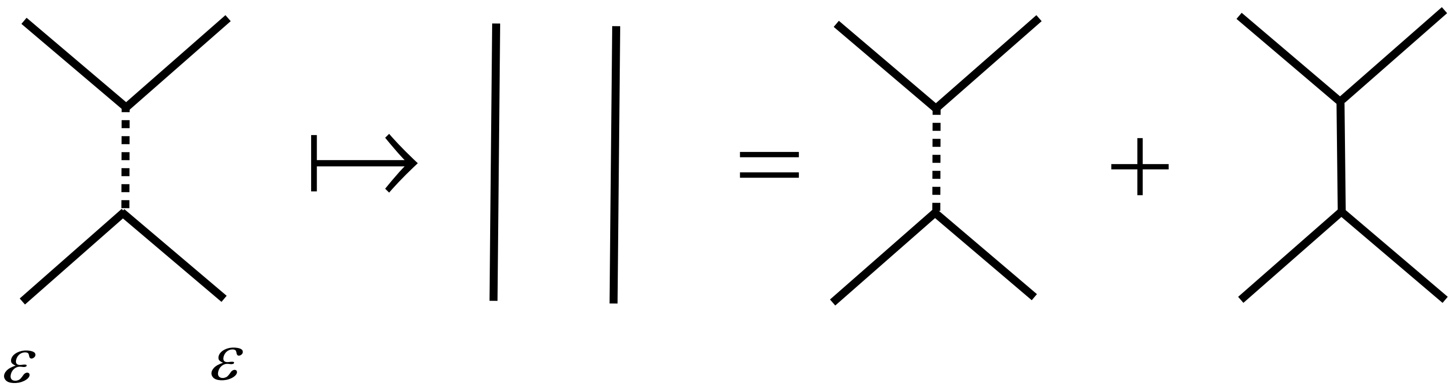
Ising:



Interferometrical Decoherence

Fibonacci:

D_{int} :



Measurement Generated Braiding!

Using Interferometric Measurements is similar but more complicated, requiring the density matrix description.
The resulting “forced measurement” procedure must include an additional measurement (of 8 or fewer anyons, i.e. still bounded size) in each teleportation attempt to ensure the overall charge of the topological qubits being acted upon remains trivial.

Note: For the Ising model TQC qubits, interferometric measurements <u>are</u> projective.

Ising vs Fibonacci

(in FQH)

Ising

- Braiding not universal
  (needs one gate supplement)
- $\Delta_{\nu=5/2} \sim 600$ mK
- Braids = Natural gates
  (braiding = Clifford group)
- No leakage from braiding
- Projective MOTQC
  (2 anyon measurements)
- Measurement difficulty distinguishing I and ψ
  (precise phase calibration)

Fibonacci

- Braiding is universal
- $\Delta_{\nu=12/5} \sim 70$ mK
- Braids = Unnatural gates
  (see Bonesteel, et. al.)
- Inherent leakage errors
  (from entangling gates)
- Interferometrical MOTQC
  (2,4,8 anyon measurements)
- Robust measurement distinguishing I and ε
  (amplitude of interference)

Conclusion

- Quantum state teleportation and entanglement resources have anyonic counterparts.
- Bounded, adaptive, non-demolitional measurements can generate the braiding transformations used in TQC.
- Stationary computational anyons hopefully makes life easier for experimental realization.
- Experimental realization of FQH double point-contact interferometers is at hand.